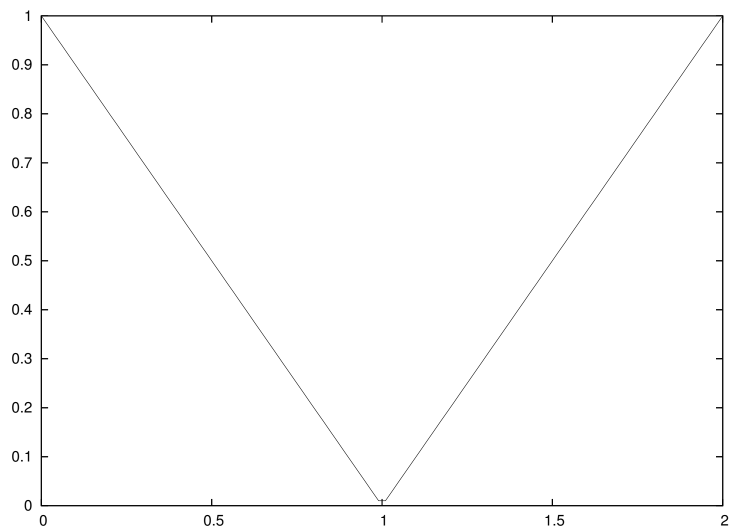
Math 116 Section 04

Quiz 3 Name ____________________
July 29, 2005
Instructor: Charles Cuell Student Number ______________

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions. Evaluate the definite integral $\int_0^2 |x-1|dx$ by interpreting it as an area.

$$|x-1| = \begin{cases} x-1 & \text{when } x \geq 1 \\ 1-x & \text{when } x < 1 \end{cases}$$

The graph is below. It is two triangles of area $\frac{1}{2}$, so that the total area is 1.



Let $g(x) = \displaystyle\int_x^{\sin(x)} \ln\left(\frac{1}{t^2+1}\right)\, dt$. What is $g'(x)$?

To use the fundamental theorem of calculus, the integral must be rewritten. For any number a,

$$\begin{aligned}
\int_x^{\sin(x)} \ln\left(\frac{1}{t^2+1}\right) dt &= \int_x^{a} \ln\left(\frac{1}{t^2+1}\right) dt + \int_a^{\sin(x)} \ln\left(\frac{1}{t^2+1}\right) dt \\
&= -\int_a^{x} \ln\left(\frac{1}{t^2+1}\right) dt + \int_a^{\sin(x)} \ln\left(\frac{1}{t^2+1}\right) dt
\end{aligned}$$

Then,

$$\begin{aligned}
g'(x) &= \frac{d}{dx}\left(-\int_a^{x} \ln\left(\frac{1}{t^2+1}\right) dt\right) + \frac{d}{dx}\int_a^{\sin(x)} \ln\left(\frac{1}{t^2+1}\right) dt \\
&= -\ln\left(\frac{1}{x^2+1}\right) + \ln\left(\frac{1}{\sin^2(x)+1}\right)\frac{d}{dx}\sin(x) \\
&= -\ln\left(\frac{1}{x^2+1}\right) + \ln\left(\frac{1}{\sin^2(x)+1}\right)\cos(x)
\end{aligned}$$